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Timeless formulation of Wigner's friend scenarios

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Hong Kong, the Quantum Information Structure of Space-Time,
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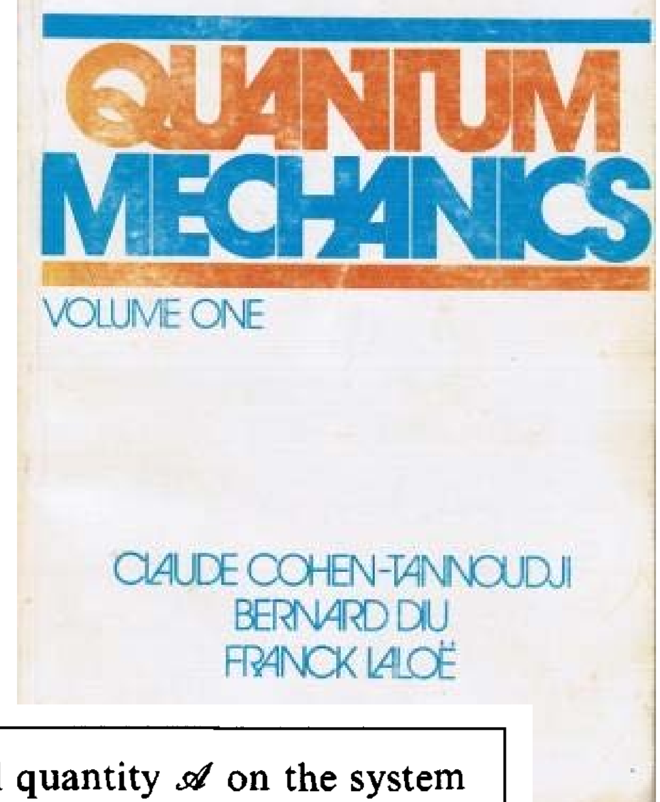
Outlook

- The quantum measurement problem
- Wigner‘s friend gedanken experiment
- Page-Wootters timeless formalism
- Conditional probabilities in Wigner‘s friend scenarios
- What is the “true” quantum state? Is the collapse of the wave function absolute or relative?



The postulates of quantum mechanics

…

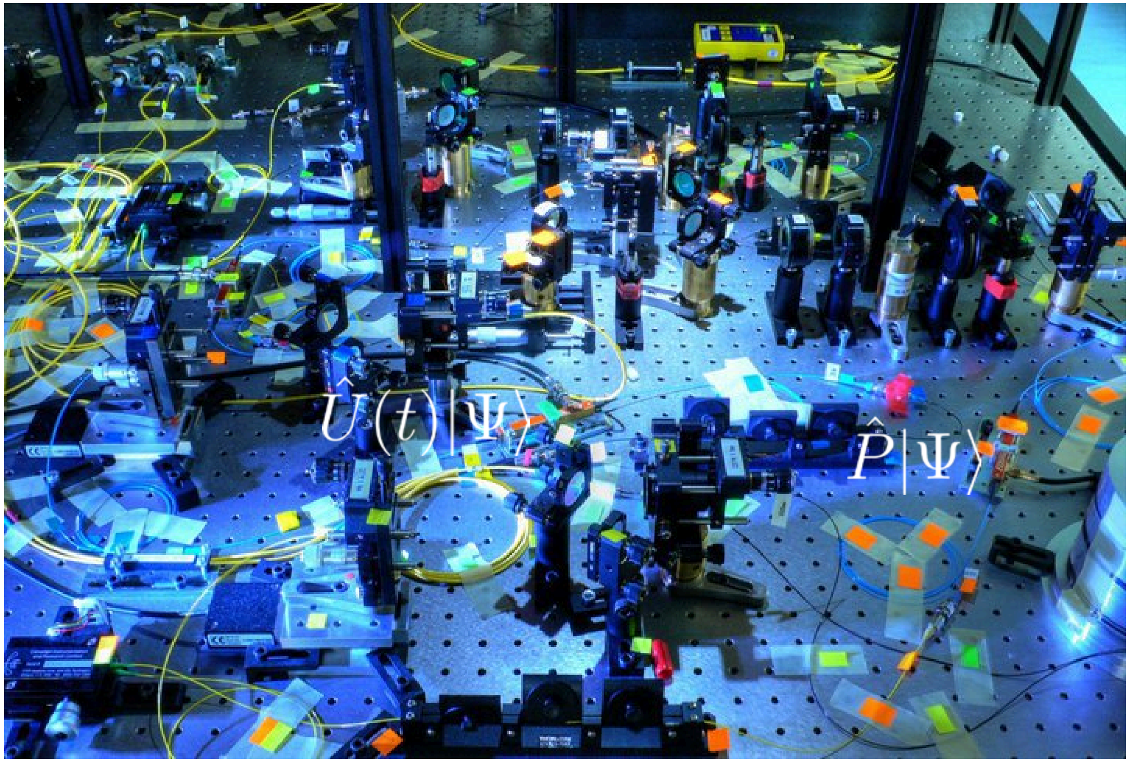
Fifth Postulate: If the measurement of the physical quantity $\mathscr{A}$ on the system in the state $|\psi\rangle$ gives the result a_n, the state of the system immediately after the measurement is the normalized projection, $\dfrac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$, of $|\psi\rangle$ onto the eigensubspace associated with a_n.

Sixth Postulate: The time evolution of the state vector $|\psi(t)\rangle$ is governed by the Schrödinger equation:

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

where $H(t)$ is the observable associated with the total energy of the system.

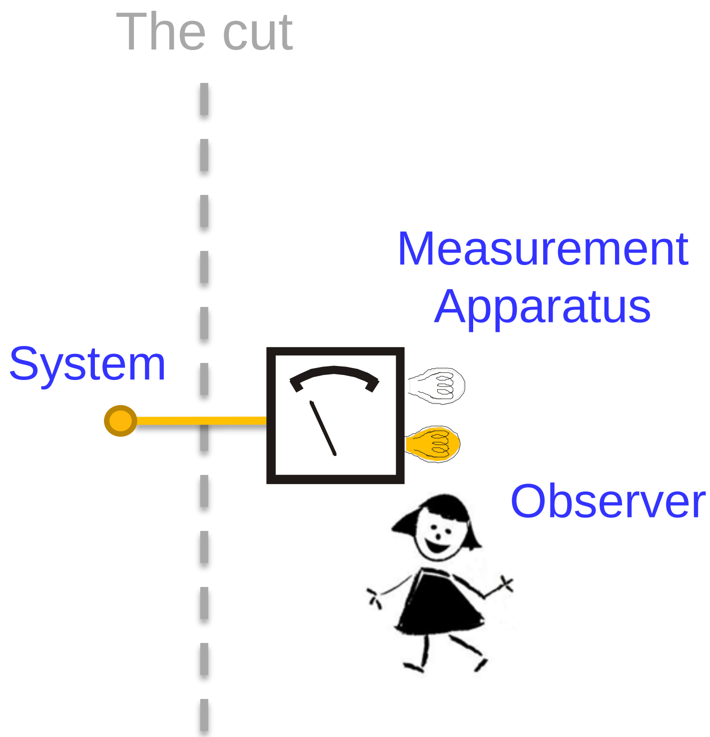
The quantum measurement problem



What makes a measurement a measurement? When to choose to apply unitary evolution and when the projection rule?

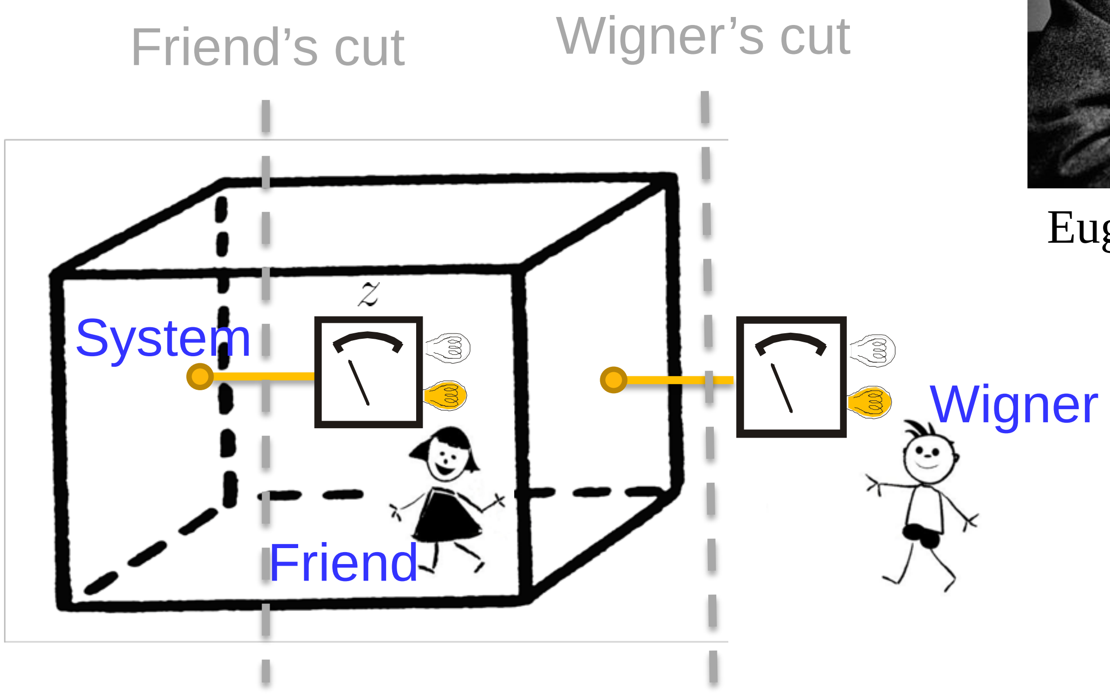
(Presumably, manufacturers of photo-diodes know the answer.)

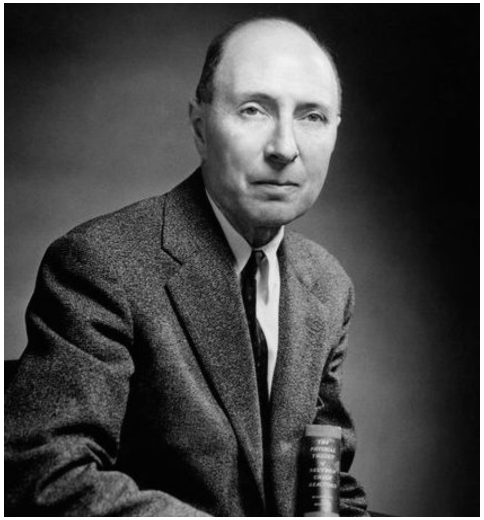
The “cut”



The „cut“ as a *functional* distinction between object and subject, not a physical one between „microworld“ and „macroworld“.

The "cut"



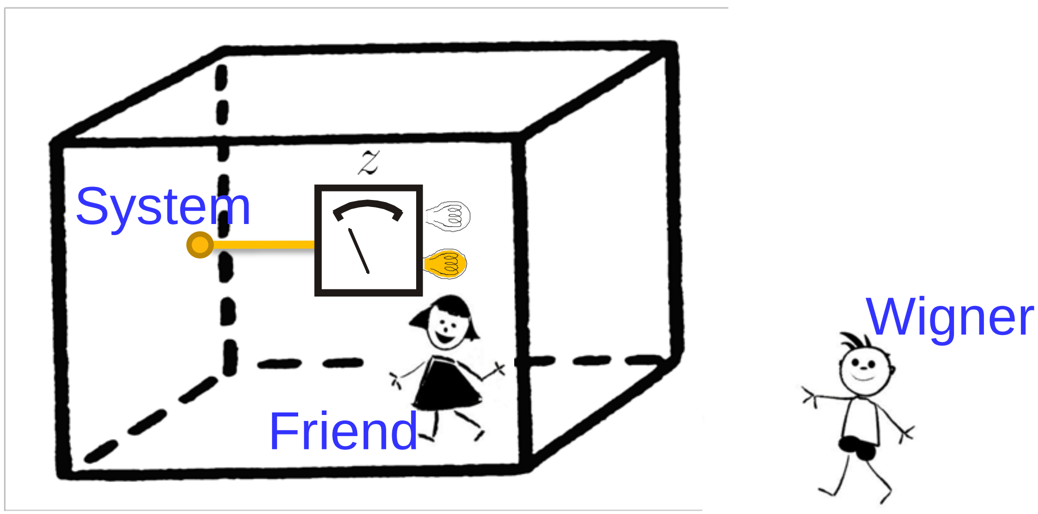


Eugene Wigner, 1961

In moving the "cut", the F's measurement instrument loses its function and becomes itself a quantum system – an object that can be observed by a further set of W's measurement instruments.

Wigner's friend thought experiment

While F performs a measurement on the system and subsequently applies the state-update rule, W describes the entire process unitarily.



$$\text{Wigner:} \quad |\psi(0)\rangle = 1/\sqrt{2}(|\uparrow\rangle_S + |\downarrow\rangle_S)|R\rangle_F$$

$$|\psi(t)\rangle = 1/\sqrt{2}(|\uparrow\rangle_S |\uparrow\rangle_F + |\downarrow\rangle_S |\downarrow\rangle_F)$$

$$\text{Friend:} \quad \text{Either } |\uparrow\rangle_S \text{ or } |\downarrow\rangle_S$$

All degrees of freedom that get coupled with the outcomes (apparatus, friend's memory, etc.)

What is the “true” quantum state?

There is nothing wrong with W and F assigning different quantum states to their respective experimental situations.

The probabilities are to be understood as **relational in the sense that their determinacy is relative to an observer** *(no-go theorem for “observation-independent facts”).*

(*Rovelli’s relational interpretation*, *QBism*, *Neo-Copenhagen interpretation*, *Many-worlds etc.)*

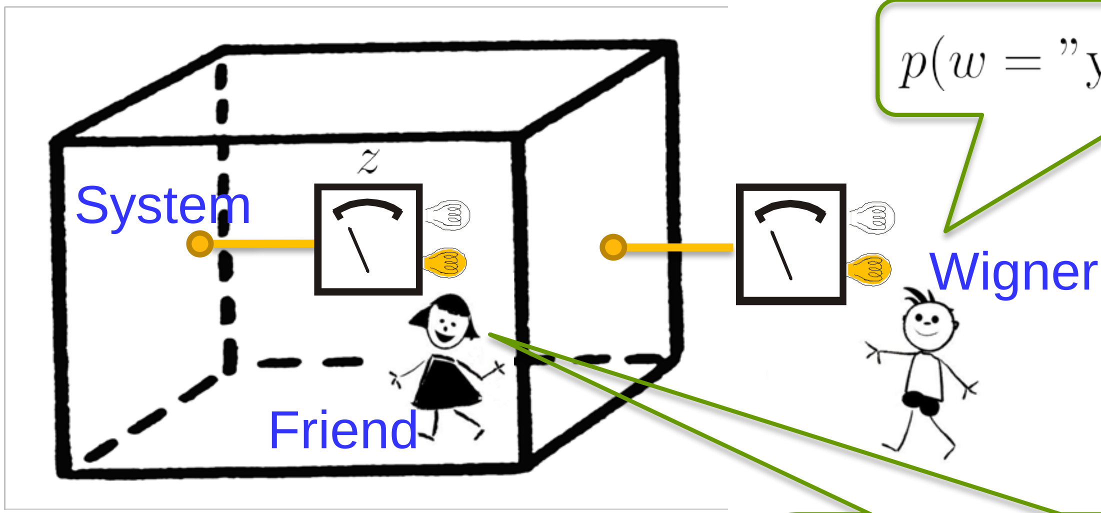
Can we think of an experimental situation in which W or F might get an evidence that their state assignment is “incomplete”?

Č.B., arXiv:1507.05255, arXiv:1804.00749

F's prediction about W's measurement

W verifies his state assignment by performing the measurement:

$$\{\text{“yes”}:\ |\psi(t)\rangle\langle\psi(t)|,\ \text{“no”}:\ \mathbb{1} - |\psi(t)\rangle\langle\psi(t)|\}$$



$$p(w = \text{”yes”}|f = \uparrow) = 1(?)$$

$$|\psi(t)\rangle = 1/\sqrt{2}(|\uparrow\rangle_S|\uparrow\rangle_F + |\downarrow\rangle_S|\downarrow\rangle_F)$$

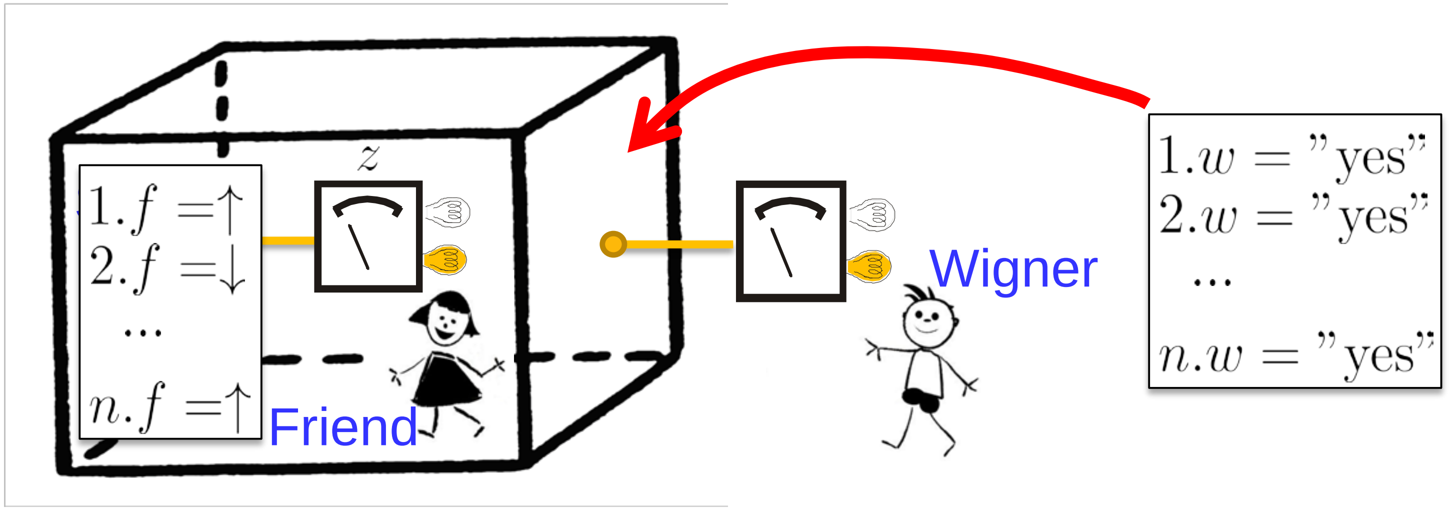
1. QM makes no predictions for measurements on the observer (?)
2. $p(w = \text{”yes”}|f = \uparrow) = 1/2\ (?)$
3. $p(w = \text{”yes”}|f = \uparrow) = 1\ (?)$

$$p(w = \text{”yes”}|f = \uparrow) = ?$$

F's prediction about W's measurement

W verifies his state assignment by performing the measurement:

$$\{\text{“yes”}:\ |\psi(t)\rangle\langle\psi(t)|,\ \text{“no”}:\ \mathbb{1}-|\psi(t)\rangle\langle\psi(t)|\}$$



$$p(w=\text{”yes”}\,|\,f=\uparrow)=1$$

The state is an eigenstate of the measurement operator, and hence the measurement can be performed repeatedly without „disturbing“ it.

Veronika Baumann, Č.B., arXiv:1901.11274

It seems that sometimes F‘s should adopt W‘s quantum state assignment to make her predictions.

When to use one or the other state? Is there a quantum formalism that would enable F (and W) to compute conditional probabilities in a general situation?

Page-Wootters timeless formalism

In 1983 Don Page and William Wootters (PW) suggested a formalism to address the “problem of time”. The Hamiltonian constraint:

$$\hat{H}|\Psi\rangle\rangle = 0 \;\Rightarrow i\hbar\frac{d|\Psi\rangle\rangle}{dt} = \hat{H}|\Psi\rangle\rangle = 0$$

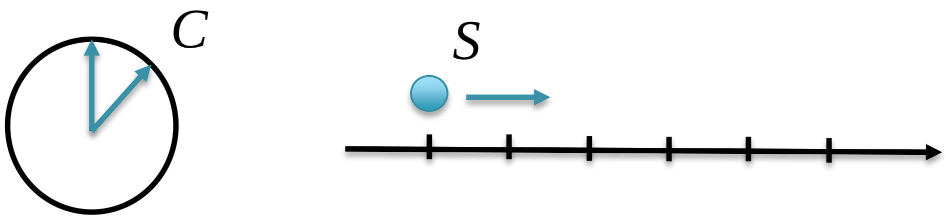
PW formalism assigns one **timeless state** from which probabilities can be computed, without the need to evolve quantum states at and in-between measurements.

Idea: Apply it to Wigner’s friend scenarios to overcome the dichotomy between the unitary evolution and the state-update rule.

Page-Wootters timeless formalism

The Hamiltonian constraint:

$$\hat{H}|\Psi\rangle\rangle = \left(\hat{H}_C + \hat{H}_S\right)|\Psi\rangle\rangle = 0$$



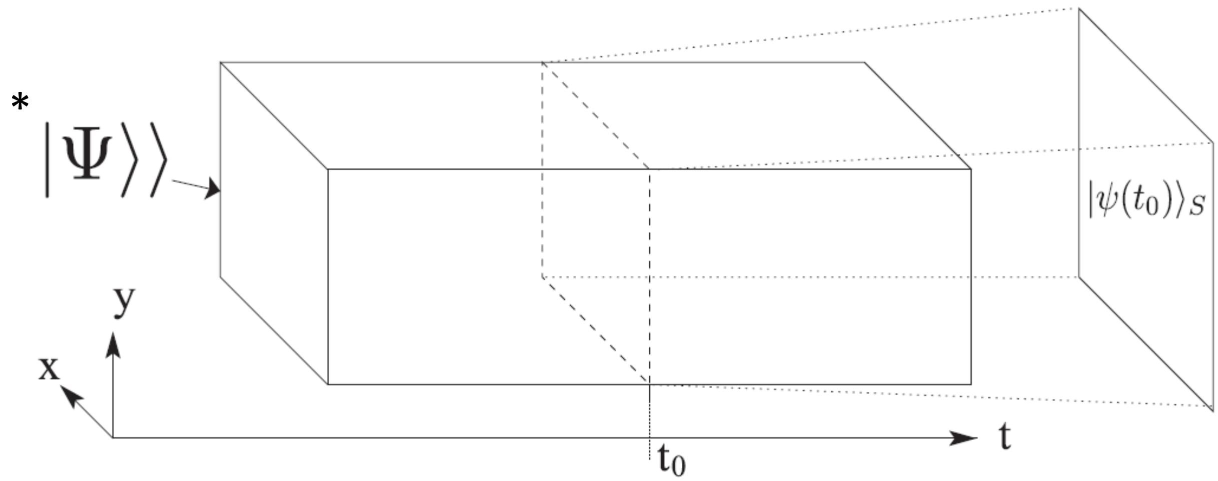
Kinematical Hilbert space: $|\psi\rangle \in \mathcal{H}_C \otimes \mathcal{H}_S$

Physical Hilbert space: $|\Psi\rangle\rangle = P^{\mathrm{ph}}|\psi\rangle \in \mathcal{H}, \;\; P^{\mathrm{ph}} := \int_{\mathbb{R}} ds\, e^{-is\hat{H}}$

Clock state indicating time *t*

$$|t'\rangle = e^{-i\hat{H}_C(t'-t)}|t\rangle, \;\; \langle t'|t\rangle = \delta(t'-t), \;\; \mathbb{1}_C = \int dt|t\rangle\langle t|$$

Timeless quantum state



$$|\Psi\rangle\rangle = \left(\int_{\mathbb{R}} |t\rangle\langle t|\right) \otimes \mathbb{1}_S |\Psi\rangle\rangle = \int_{\mathbb{R}} dt\, |t\rangle |\psi_S(t)\rangle$$

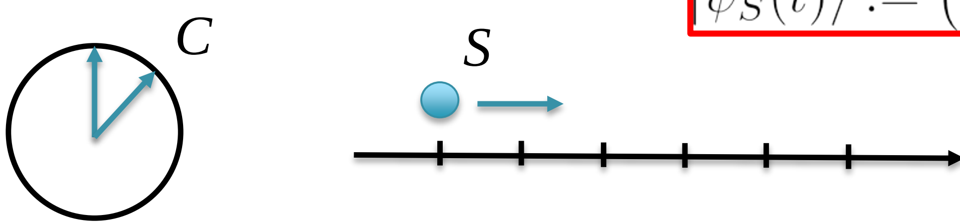
Superposition of „histories“

* Taken from V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. D 92, 045033 (2015)

Time is what a clock reads

„The state of the system at the time t“ is the joint state $|\Psi\rangle\rangle$ of the clock and the system, condition on the clock beeing in state $|t\rangle$.

$$\boxed{|\psi_S(t)\rangle := \left(\langle t| \otimes \mathbb{1}_S\right)|\Psi\rangle\rangle.}$$



The state satisfies the Schrödinger equation: $i\frac{d}{dt}|\psi_S(t)\rangle = \hat{H}_S|\psi_S(t)\rangle$

D. N. Page and W. K. Wootters, Phys. Rev. D 27, 2885 (1983)

Born’s rule

The single-time probability:

$$P\,(m \text{ when } t) = \frac{\langle\langle\Psi|\big(|t\rangle\langle t|\otimes\Pi_m\big)|\Psi\rangle\rangle}{\langle\langle\Psi|\big(|t\rangle\langle t|\otimes\mathbb{1}_S\big)|\Psi\rangle\rangle}$$

Mesurement operator

$\hat{M} = \sum_m m\Pi_m$

A problem with **the conditional probability** (Kuchar‘s criticism):

$$P(m \text{ when } t_2|n \text{ when } t_1)$$

$$= \frac{P(m \text{ when } t_2 \;\&\; n \text{ when } t_1)}{P(n \text{ when } t_1)}$$

$$= \frac{\langle\langle\Psi|\big(|t_1\rangle\langle t_1|\otimes\Pi_n\big)\big(|t_2\rangle\langle t_2|\otimes\Pi_m\big)\big(|t_1\rangle\langle t_1|\otimes\Pi_n\big)|\Psi\rangle\rangle}{\langle\langle\Psi|\big(|t_1\rangle\langle t_1|\otimes\Pi_n\big)|\Psi\rangle\rangle}$$

$$= \delta^2(t_2-t_1)|\langle m|n\rangle|^2 \neq |\langle m|U(t_2-t_1)|n\rangle|^2$$

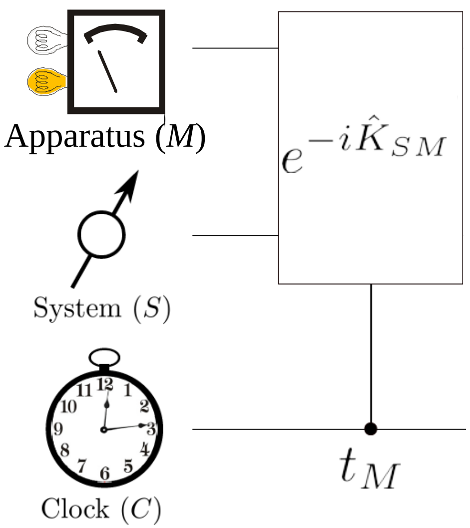
K. V. Kuchar, Int. J. Mod. Phys. D 20, 3 (2011)

Overcoming the problem

Including the interaction with the measurement apparatus in the Hamiltonian constraint

$$\hat{H}|\Psi\rangle\rangle = \left(\hat{H}_C + \hat{H}_S + \delta(\hat{T} - t_M)\hat{K}_{SM}\right)|\Psi\rangle\rangle = 0$$

$$e^{-i\hat{K}_{SM}}|\psi_S\rangle|R_M\rangle = \sum_m \Pi_m|\psi_S\rangle|m_M\rangle$$



$$\boxed{\begin{aligned} |\Psi\rangle\rangle &= \int_{-\infty}^{t_M} dt\,|t\rangle|\psi_S\rangle|R_M\rangle \\ &+ \int_{t_M}^{\infty} dt\,|t\rangle\sum_m \Pi_m|\psi_S\rangle|m_M\rangle \end{aligned}}$$

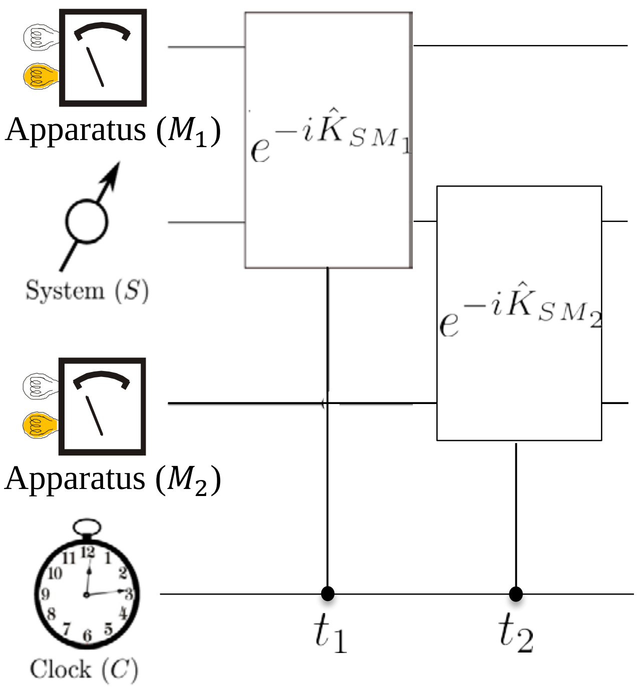
$$P\,(m \text{ when } t) = \frac{\langle\langle\Psi|\left(|t\rangle\langle t| \otimes \mathbb{1}_S \otimes \Pi^m\right)|\Psi\rangle\rangle}{\langle\langle\Psi|\left(|t\rangle\langle t| \otimes \mathbb{1}_{SM}\right)|\Psi\rangle\rangle}$$

V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. D 92, 045033 (2015)

Overcoming the problem

Including the interaction with the measurement apparatuses in the Hamiltonian constraint

$$\hat{H}|\Psi\rangle\rangle = \left(\hat{H}_C + \hat{H}_S + \delta(\hat{T}-t_1)\hat{K}_{SM_1} + \delta(\hat{T}-t_2)\hat{K}_{SM_2}\right)|\Psi\rangle\rangle = 0$$

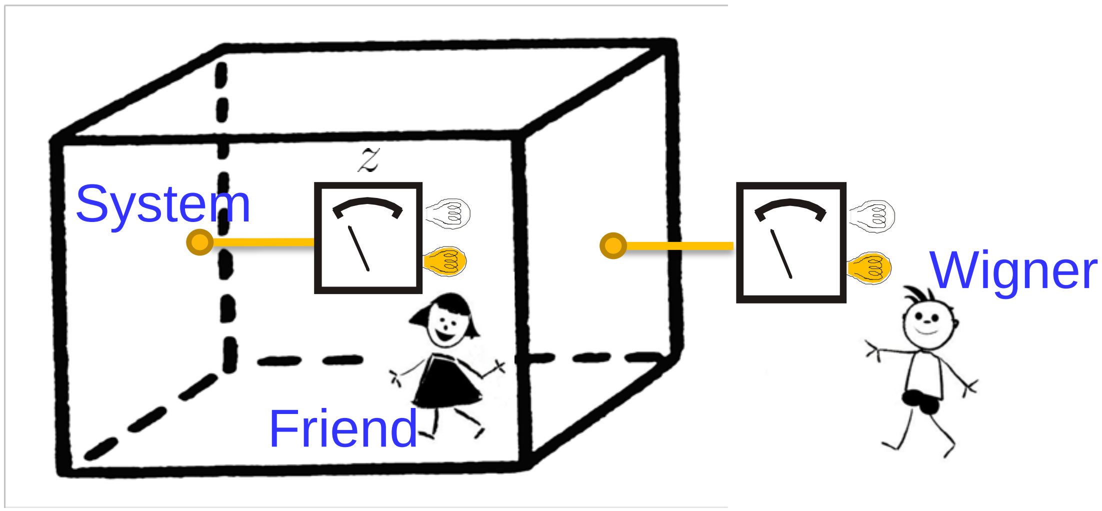


$$\boxed{\begin{aligned} |\Psi\rangle\rangle \quad &= \int_{-\infty}^{t_1} dt\, |t\rangle|\psi_S\rangle|R_{M_1}\rangle|R_{M_2}\rangle \\ &+ \int_{t_1}^{t_2} dt\, |t\rangle \sum_m \Pi_m|\psi_S\rangle|m_{M_1}\rangle|R_{M_2}\rangle \\ &+ \int_{t_2}^{\infty} dt\, |t\rangle \sum_{n,m} \Pi_n\Pi_m|\psi_S\rangle|m_{M_1}\rangle|n_{M_2}\rangle \end{aligned}}$$

$$P(n \text{ when } t_2 | m \text{ when } t_1) = \frac{\langle\langle\Psi|\big(|t_2\rangle\langle t_2| \otimes \mathbb{1}_S \otimes \Pi^m \otimes \Pi^n\big)|\Psi\rangle\rangle}{\langle\langle\Psi|\big(|t_1\rangle\langle t_1| \otimes \mathbb{1}_S \otimes \Pi^m \otimes \mathbb{1}_{M_2}\big)|\Psi\rangle\rangle}$$

V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. D 92, 045033 (2015)

Wigner's friend situation



Initial state: $|\psi_S\rangle = a|\uparrow_S\rangle + b|\downarrow_S\rangle,\ a, b \in \mathbb{R}$

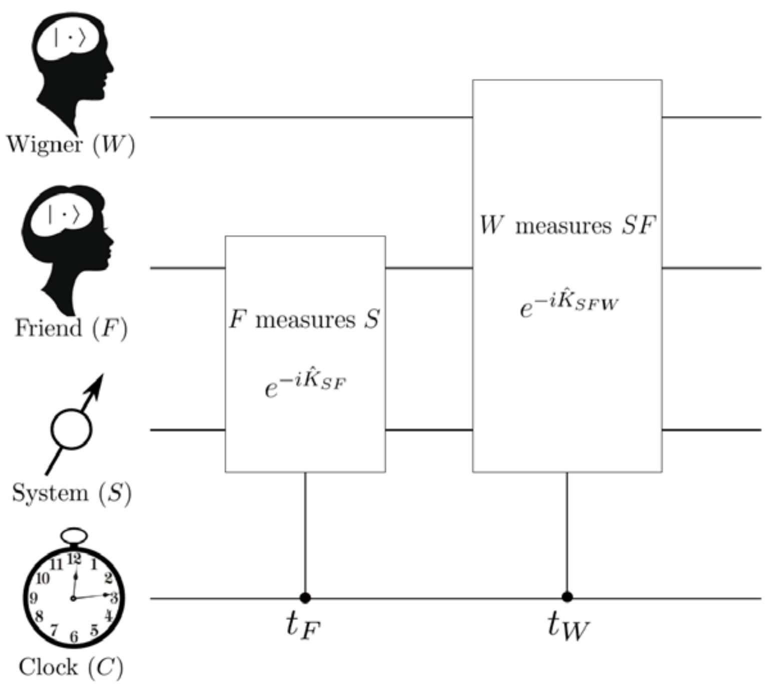
The friend measures the spin along *z*

Wigner measures $\Pi_{yes} = |\text{yes}\rangle\langle\text{yes}|$ and $\Pi_{no} = \mathbb{1} - |\text{yes}\rangle\langle\text{yes}|$

$$|\text{yes}_{SF}\rangle = \alpha|\uparrow_S\rangle|\uparrow_F\rangle + \beta|\downarrow_S\rangle|\downarrow_F\rangle,\ \alpha, \beta \in \mathbb{R}$$

Wigner's friend situation

$$\hat{H}|\Psi\rangle\rangle = \left(\hat{H}_C + \hat{H}_S + \delta(\hat{T} - t_F)\hat{K}_{SF} + \delta(\hat{T} - t_W)\hat{K}_{SFW}\right)|\Psi\rangle\rangle = 0$$



$$\boxed{\begin{aligned} |\Psi\rangle\rangle &= \int_{-\infty}^{t_F} dt\,|t\rangle|\psi_S\rangle|R_F\rangle|R_W\rangle \\ &+ \int_{t_F}^{t_W} dt\,|t\rangle \sum_{f\in\{\uparrow,\downarrow\}} \Pi_f|\psi_S\rangle|f_F\rangle|R_W\rangle \\ &+ \int_{t_W}^{\infty} dt\,|t\rangle \sum_{\substack{f\in\{\uparrow,\downarrow\} \\ w\in\{\text{yes, no}\}}} \Pi_w\Pi_f|\psi_S\rangle|f_F\rangle|w_W\rangle \end{aligned}}$$

“Collapse” conditional probability

Definition:

Applied s.t. the state still satisfies the constraint (Dolby, gr-qc/0406034)

$$P_1\left(n \text{ when } t_2 \,|\, m \text{ when } t_1\right) = \frac{\langle\langle\Psi|\,|t_1\rangle\langle t_1| \otimes \Pi^m P^{\mathrm{ph}} \left(|t_2\rangle\langle t_2| \otimes \Pi^n\right) P^{\mathrm{ph}} |t_1\rangle\langle t_1| \otimes \Pi^m |\Psi\rangle\rangle}{\langle\langle\Psi|\,|t_1\rangle\langle t_1| \otimes \Pi^m |\Psi\rangle\rangle}$$

V. Baumann, F. Del Santo, A. R. H. Smith, F. Giacomini, E. Castro-Ruiz, and C. Brukner, arXiv:1911.09696

“Collapse” conditional probability

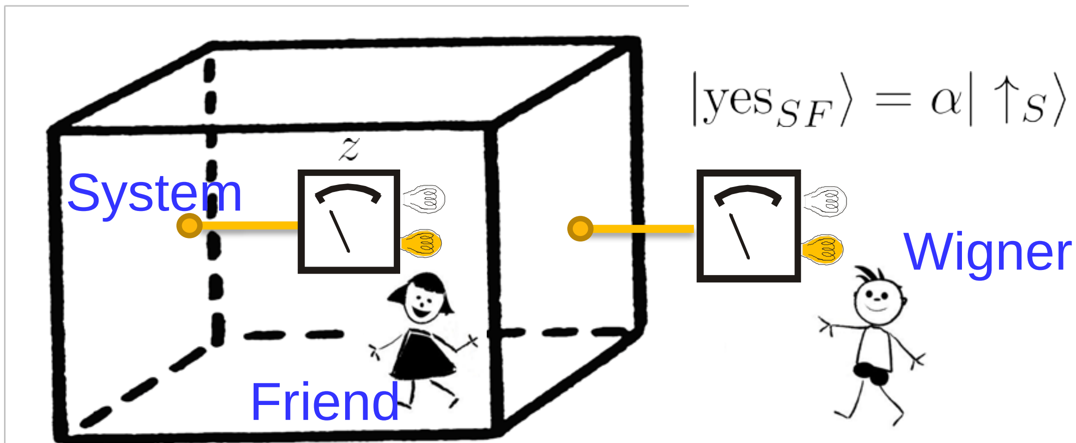
Definition:

$$P_1\left(n \text{ when } t_2 \,\middle|\, m \text{ when } t_1\right) = \frac{\langle\langle\Psi||t_1\rangle\langle t_1|\otimes\Pi^m P^{\text{ph}}(|t_2\rangle\langle t_2|\otimes\Pi^n)P^{\text{ph}}|t_1\rangle\langle t_1|\otimes\Pi^m|\Psi\rangle\rangle}{\langle\langle\Psi||t_1\rangle\langle t_1|\otimes\Pi^m|\Psi\rangle\rangle}$$

- It is always a **well-defined probability**
- Correspond to **applying the “state-update rule”** after every measurement
- Reduce to the standard probability rule for non-Wigner’s friend scenarios

V. Baumann, F. Del Santo, A. R. H. Smith, F. Giacomini, E. Castro-Ruiz, and C. Brukner, arXiv:1911.09696

“Collapse” conditional probability



$$|\mathrm{yes}_{SF}\rangle = \alpha|\uparrow_S\rangle|\uparrow_F\rangle + \beta|\downarrow_S\rangle|\downarrow_F\rangle$$

Either $|\uparrow_S\rangle|\uparrow_F\rangle$ or $|\downarrow_S\rangle|\downarrow_F\rangle$

$P_1\,(w \text{ when } t_2 \,|\, f \text{ when } t_1)$

f \ w	yes	no
$\uparrow$	α^2	β^2
$\downarrow$	β^2	α^2

$P_1\,(f \text{ when } t_1 \,|\, w \text{ when } t_2)$

f \ w	yes	no
$\uparrow$	α^2	β^2
$\downarrow$	β^2	α^2

V. Baumann, F. Del Santo, A. R. H. Smith, F. Giacomini, E. Castro-Ruiz, and C. Brukner, arXiv:1911.09696

“Unitary” conditional probability

Definition:

$$\boxed{\begin{aligned} &P_1\left(n \text{ when } t_2 \,\middle|\, m \text{ when } t_1\right) \\ &\quad = \frac{\langle\langle\Psi|(|t_2\rangle\langle t_2|\otimes\Pi^n)P^{\rm ph}(|t_1\rangle\langle t_1|\otimes\Pi^m)|\Psi\rangle\rangle}{\langle\langle\Psi||t_1\rangle\langle t_1|\otimes\Pi^m|\Psi\rangle\rangle} \end{aligned}}$$

- This is a **well-defined probability only when measurement operators commute** on the physical state when compared at the same instant of time

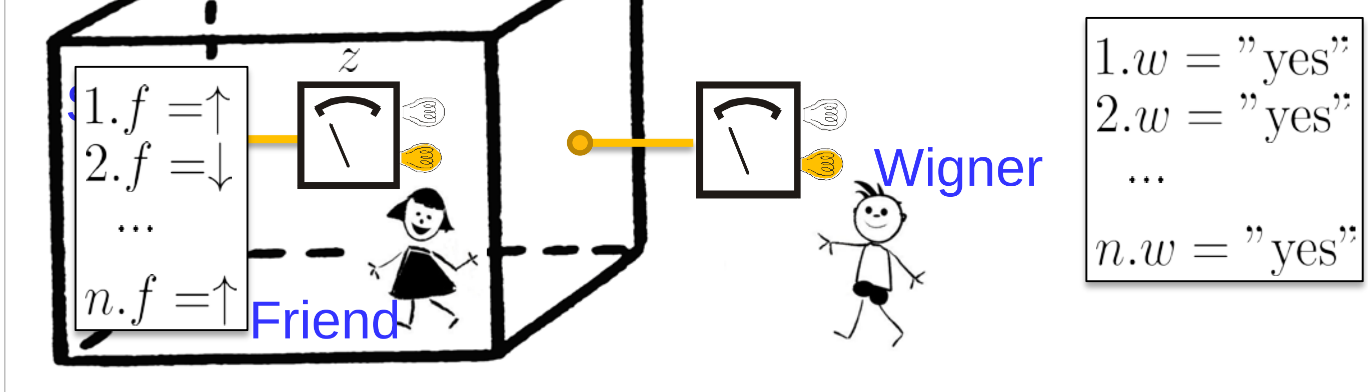
$$[U(t_1,t_2)\Pi^m U^\dagger(t_1,t_2),\Pi^n]|\Psi\rangle\rangle = 0$$

The case of **non-disturbance:** $a = \alpha$ and $b = \beta$!

- Reduce to the standard probability rule for non-Wigner’s friend scenarios

“Unitary” conditional probability

$$|\text{yes}_{SF}\rangle = \alpha|\uparrow_S\rangle|\uparrow_F\rangle + \beta|\downarrow_S\rangle|\downarrow_F\rangle$$



$$|\psi(t)_{SF}\rangle = \alpha|\uparrow_S\rangle|\uparrow_F\rangle + \beta|\downarrow_S\rangle|\downarrow_F\rangle$$

$P_3\,(w$ when $t_2\,|\,f$ when $t_1)$

f \ w	yes	no
$\uparrow$	1	0
$\downarrow$	1	0

$P_3\,(f$ when $t_1\,|\,w$ when $t_2)$

f \ w	yes	no
$\uparrow$	α^2	0
$\downarrow$	β^2	0

V. Baumann, F. Del Santo, A. R. H. Smith, F. Giacomini, E. Castro-Ruiz, and C. Brukner, arXiv:1911.09696

Conclusions

- Several definitions for the conditional probabilities for Wigner's friend scenarios (Page-Wootters formalism)
- All of them are equivalent for standard, non-Wigner's friend scenarios
- There are more than just the two probability rules. Is there a way of classifying all of them? What is their operational meaning? Can one reject some (all?) of them on the basis of logical inconsistences?
- Can one construct maps between the perspectives of Wigner and his friend?

THANK YOU!

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